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Manuscript received October 14, 1963; revision received March 16, 1964; paper accepted March 18, 1964.

Thickness Distribution in a Sheet Formed by Impinging Jets

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The impinging jets atomizer possesses certain advantages as a spraying device in prime movers, particularly for propellant injection in liquid rocket combustors (*1, 2*). As in other spraying devices the drop size characteristics are closely related to the thickness of the spray sheet prior to breakup. Study of the thickness distribution in the sheet formed by impinging jets is also of importance for the design of fan spray nozzles, which operate on a closely similar principle (*3*).

Though several attempts (*4, 5, 6*) have been made to analyze the thickness distribution in the sheet formed by impinging jets, no satisfactory solution to this problem has hitherto been available. This paper provides the first analytical solution fitting all experimental data reported in the literature.

STREAMLINE PATTERN OF FLOW

Consider two cylindrical and equal jets of radius R flowing at equal velocity v_0 but in opposite directions (Figure 1). If the two jets collide obliquely at a total included angle of 2θ, it is found that the liquid spreads into a flat sheet flowing in a plane perpendicular to that containing the axes of the two jets.

The flow around the sheet is uneven, the flux being largest in the forward flow direction and smallest in the

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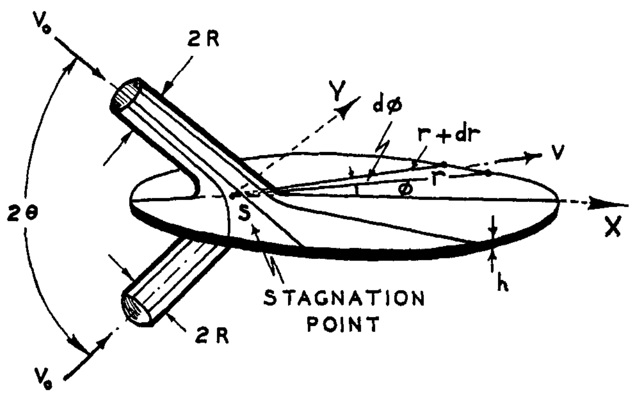

Fig. 1. Equal thickness contour of sheet formed by impinging jets.

backward flow direction. The problem is to find the variation in sheet thickness h with radial distance r and angular position ϕ.

Following previous analyses (*4, 5, 6*) the authors examine the case of steady state flow with gravity, surface tension, and viscosity effects ignored. Such conditions can be closely realized experimentally.

Consider a plausible streamline pattern of this ideal fluid system. Because symmetry exists about the center plane of the sheet, one need only refer to one of the jets.

Upstream of the impingement region the static pressure is assumed equal throughout the jet, coinciding with the pressure of the surroundings. The streamlines are therefore straight lines parallel to the jet axis, with a constant flow velocity. Downstream of the impingement region the static pressure is similarly equal throughout the sheet. The streamlines spread radially in all directions from some stagnation point S situated in the impingement region.

The only location in the system in which a force is exerted on the liquid is the impingement zone. This force, generated by the destruction of the vertical components of the momentum contained in the respective jets, is the agent which deflects each streamline of the jet from its straight line course into the plane of the sheet.

In passing through the impingement zone the streamlines of the jet are accelerated away from the stagnation point S. Figure 2 shows the flow pattern in a vertical section of the system, taken in a plane passing through S and cutting the sheet at the angular positions $\phi = 0$ and $\phi = \pi$.

The only streamline of the jet suffering no deflection is the separation streamline* PS which intersects the center plane of the sheet at the stagnation point S. It separates streamlines of the jet flowing into the sheet section in the direction of $\phi = 0$ from those flowing into the opposite sheet section $\phi = \pi$.

Each streamline of the jet is seen to retain the same angular direction in the sheet (at either $\phi = 0$ or $\phi = \pi$), acquiring no rotational velocity component about the sheet axis. This property follows from the assumption of irrotational flow of the ideal fluid.

By applying similar considerations to each section of the jet containing the separation streamline the flow in the jet can be related to the flow in the sheet.

RELATION OF THE FLOW IN THE JET TO THE FLOW IN THE SHEET

From analytical geometry a section through the jet in a plane parallel to the sheet is an ellipse having a major axis equal to $2R/\sin\theta$ and a minor axis equal to $2R$ (Figure 3).

The flow pattern in the system described above implies that mass flow rate is conserved within angular differential elements taken relative to the separation streamline (Figure 2). Thus for a material balance the origin of the coordinates in the jet must be taken at the separation point P, which is located at the intersection of the separation streamline with the plane of the ellipse.

The origin for the flow in the sheet is at the stagnation point S. Hence a material balance equating input of material flowing in an angular element of the ellipse between ϕ and $\phi + d\phi$ to output in the corresponding element of the sheet (Figures 1 and 3) is given by

$$(q\, d\phi)\ (q/2)\ (v_0 \sin\theta) = (r\, d\phi)\ (h/2)\ (v) \qquad (1)$$

If r, the radial distance in the sheet, is sufficiently large, the inward velocity component of the attenuating sheet is negligibly small. From an energy balance it follows that the velocity v_0 in the jet is equal to the velocity v in the sheet. Hence Equation (1) expressed in nondimensional form becomes

$$\frac{hr}{R^2} = \frac{q^2}{R^2} \sin\theta \qquad (2)$$

It is thus seen that the thickness distribution in the sheet $(hr/R^2) = f(\phi)$ can be easily calculated from the known variation of q with ϕ, provided the location of the separation point P is known.

* The term *separation* as used here should not be confused with a similar term employed to describe a boundary-layer effect.

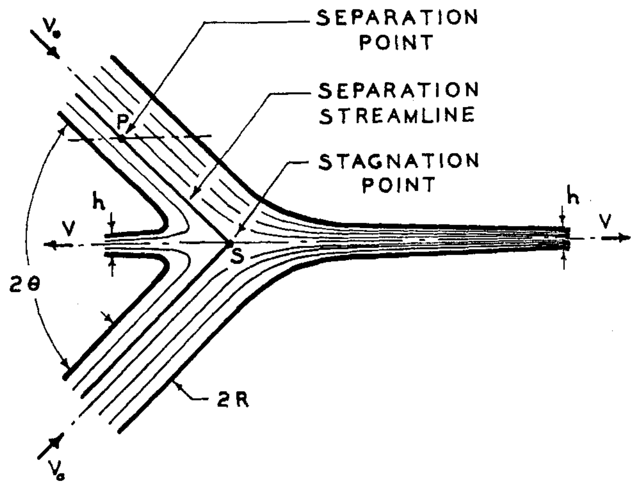

Fig. 2. Streamline pattern in a vertical section containing the separation streamlines.

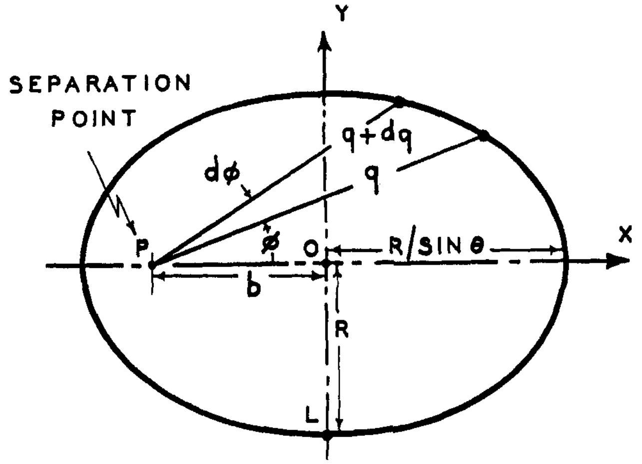

Fig. 3. Section of a jet in a plane parallel to the sheet.

DETERMINATION OF THE SEPARATION POINT

The separation point P must be situated at some point along the X axis, which constitutes an axis of symmetry of the system. The distance $OP = b$ between the separation point and the center of the ellipse (Figure 3) can be determined from a momentum balance. Equating the forward momentum (in the X direction) contained in the two jets with the total X momentum generated in the sheet one obtains

$$2\pi R^2 \rho v_o^2 \cos\theta = \int_0^{2\pi} (hr)\rho v^2 \cos\phi \, d\phi \qquad (3)$$

Substituting for (hr) from Equation (2) and simplifying one gets

$$\pi \cot\theta = \int_0^{\pi} \left(\frac{q}{R}\right)^2 \cos\phi \, d\phi \qquad (4)$$

A second functional relationship between q and ϕ, with b and θ as parametric constants, is given by the polar equation of the ellipse, namely

$$\left(\frac{q}{R}\sin\phi\right)^2 + \left(\frac{q}{R}\cos\phi - \frac{b}{R}\right)^2 \sin^2\theta = 1 \qquad (5)$$

Eliminating (q/R) in Equation (4), using Equation (5), and integrating, one finds that a simple relationship exists between the parametric constants as follows:

$$\frac{b}{R} = \cot\theta \qquad (6)$$

This result indicates that the separation point P is located at a focus of the ellipse. With reference to Figure 3, Equation (6) shows that angle $LPO = \theta$. Hence $LP = R/\sin\theta$, that is half the major axis, proving that P is a focal point.

THICKNESS DISTRIBUTION IN THE SHEET

When one combines Equations (2), (5), and (6), the desired expression for the thickness distribution in the sheet is obtained:

$$\frac{hr}{R^2} = \frac{\sin^3\theta}{(1 - \cos\phi \, \cos\theta)^2} \qquad (7)$$

Figure 4 shows that the variation in the thickness parameter predicted by Equation (7) is in good agreement with Taylor's experimental data (6) taken at $\theta =$ 60, 45, and 30 deg., respectively, and with Miller's experimental results (5) obtained at $\theta = 60$ deg. In Figure 4 Taylor's measurements, which he reported dimensionally as $hr = f(\phi)$, have been converted to give the nondimensional group hr/R^2. Since Taylor used sharp edged orifices, also taking care to ensure laminar conditions, the jet radius R was calculated from the orifice dimensions with a value of 0.72 for the coefficient of contraction (7).

It is of interest to compare Equation (7) with the following semiempirical expression previously proposed by Miller (5):

$$\frac{hr}{R^2} = \frac{\sin^2\theta}{1 - 2\cos\theta \, \cos\phi + \cos^2\theta} \qquad (8)$$

At sufficiently large values of θ, $\cos^2\theta << \cos\theta$ and $\sin^2\theta \simeq \sin^3\theta$ so that the difference between results calculated in accordance with Equations (7) and (8), respectively, is negligibly small. However this difference increases as θ is reduced and is appreciable at impingement angles below $\theta = 60$ deg.

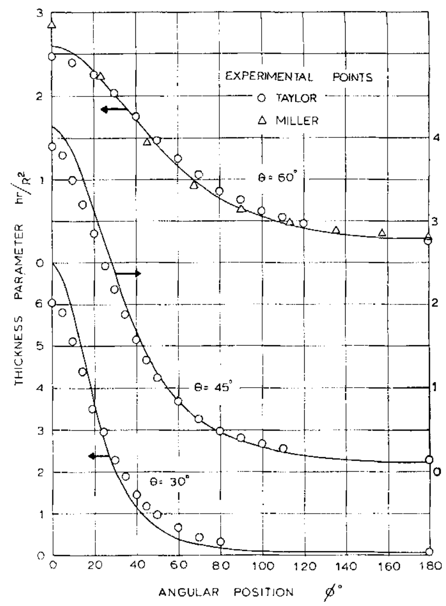

Fig. 4. Comparison of theory with experiment.

Fortuitously Equations (7) and (8) predict identical results for the ratio of backward to forward flux ($\phi = \pi$ and 0, respectively), namely

$$\frac{(hr)_\pi}{(hr)_0} = \left(\frac{1 - \cos\theta}{1 + \cos\theta}\right)^2 \qquad (9)$$

Miller (5) presented data showing good agreement with values predicted by Equation (9). This agreement provides additional experimental support to the analytical solution derived in the present paper.

NOTATION

- b = distance between the separation point and the center in the elliptical section of the jet
- h = sheet thickness
- q = polar radius of the ellipse, referred to the separation point as the pole
- r = radial distance in the sheet, measured from the stagnation point
- R = jet radius
- v = liquid velocity in the sheet
- v_o = liquid velocity in the jet
- x, y = cartesian coordinates, having their origin at the center of the jet section; the X axis is in the forward flow direction of the jets
- ϕ = angular position, referred to a pole situated along the separation streamline
- ρ = liquid density
- θ = half the total impingement angle

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Manuscript received July 17, 1963; revision received March 5, 1964; paper accepted March 5, 1964.